

# Seminar On Applied Analysis

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## Period 1 Preliminaries

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Time	Topic	Director
9.11	Convex Analysis	Yiping Lu
9.18	PDE	Yiping Lu
9.25	Numerical PDE	Yiping Lu
10.9	Opimal Control	Yixuan Wang

## Week1

### convex analysis

**Reference:** *Ivar Ekeland, Roger Teman* Convex analysis and variational problems

Convex Analysis: Convex set and function, Conjugate and subdifferentiable, proximal operator

Variation problems and Euler-Lagrange Equation

Optimal transport: Monge problem and Kantorovich Dual

## Week2

### Basic PDE

**Reference:** *L.C.Evans* Partial Differential Equation.

PDEs from Variational Problems

Method of characteristic

Fourier Transform/Laplace transform and application in PDE

Fundamental solution

Fourier Method, Energy Method, other methods

## Week3

### Finite difference scheme

**Reference:** *John Strikwerda* Finite difference schemes and partial differential equations

Finite difference scheme for poisson equation, error analysis.

Parabolic differential equation, Explicit and Implicit

Hyperbolic differential equation, upwind scheme, CFL, Dissipation and dispersion, finite volume scheme, multistep scheme, modified scheme for advection-diffusion equation

Fourier Analysis, Von Neumann Analysis, instability and stability, order of accuracy

well-posed (John chapter 11)

\* Lax Theorem

**modified equation**

## Week4

### Optimal Control

**Reference:** *L.C.Evans* AN INTRODUCTION TO MATHEMATICAL OPTIMAL CONTROL THEORY.

### Additional Week

Papers we can discuss

- A variational perspective on accelerated methods in optimization. A. Wibisono, A. Wilson, and M. I. Jordan. *Proceedings of the National Academy of Sciences*, 133, E7351-E7358, 2016.
- W. Su, S. Boyd and E. J. Candès. A differential equation for modeling Nesterov's accelerated gradient method: theory and insights. *Journal of Machine Learning Research* 17 (153), 1--43. (This is the long form or journal version of the NIPS paper.)
- Felipe Alvarez, Jerome Bolte, and Olivier Brahic. Hessian Riemannian gradient flows in convex programming. *SIAM Journal on Control and Optimization*, 43(2):477–501, 2004
- Garvesh Raskutti and Sayan Mukherjee. The information geometry of mirror descent. *IEEE Transactions on Information Theory*, 61(3):1451–1457, 2015.

## Peroid2: Optimal Transport

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**Reference:**

- *Santambrogio F.* Optimal Transport for Applied Mathematicians
- *Villani C.* Optimal transport: old and new
- *Villani C.* Topics in optimal transportation

TOPICS	Reference	
One-dimensional issue	OT for applied math ch 2	Yuxuan Zhu
L1 and $L^\infty$ theory	OT for applied math ch 3	Jikai Hou
geometry of optimal transportation	OT for applied math ch 4	Runyu Zhang
Wasserstein space	OT for applied math ch 5	Zexing Li
Probabilities	OT for applied math ch 7	Cheng Chen
Gradient flow of Wasserstein space	OT for applied math ch 8	Shicong Cen
Other Topics And Papers		

## Period3: Stochastic Analysis

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### Stochastic Differential Equation

**Reference:**

- *Bernt Øksendal* Stochastic differential equations: an introduction with applications
- *L.C.Evans* Introduction to stochastic differential equation
- *Peter E.Koloden, Eckhard Platen* Numerical Solution of stochastic differential equation

### Information Geometry

**Reference:**Shun-ichi Amari. *Information Geometry and Its Applications*. Vol. 194. Springer, 2016.

## Period4: Advanced Topics

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### Wavelet Theory

speaker: Daozhe Lin

### Multi-scale Method

speaker: Zhihan Li, Yiping Lu