Assignment1
Convex Analysis And Variational Problems

August 25, 2017

1.(Properities About Moreau Envelope) We define

$$M_{cf}(x) = \inf_y \left\{ f(y) + \frac{1}{2c} ||y - x||^2 \right\}$$

Here are some properties:

- $M_{cf} \in C^1$ even f is not
- $M_{cf} = \left((cf)^* + (1/2) ||\cdot||^2 \right)^*$
- $\nabla M_{cf}(x) = (1/c)(x - prox_{cf}(x))$
- $prox_f(x) = \nabla M_{f^*}(x)$
- $prox_{cf}(x) = x - c\nabla M_{cf}(x)$

2.(c-concave) Suppose that c is real valued. For any $\phi : X \to \cup\{-\infty\}$, we have $\phi^{c\bar{c}}\phi$ if and only if ϕ is c-concave. In general, $\phi^{c\bar{c}}$ is the smallest c-concave function larger than ϕ

3.(Inf convolution) Inf convolution palys the role of regularization in convex analysis which defined as

$$(\phi\Box\psi)(z) = \inf_{x+x'=z} [\phi(x) + \psi(x')]$$

- $(\phi\Box\psi)^* = \phi^* + \psi^*$
- let $\psi_\epsilon(x) = |x|^2/(2\epsilon)$ and $\phi_\epsilon = \phi\Box\psi_\epsilon$. For l.s.c and bounded below by an affine function.Check taht $\phi(x) = \lim_{\epsilon\to 0} \phi_\epsilon(x)$

4.Can you prove that the the Kantorovich dual problem has the same answer as the primal one? (You can see the chapter 1.6.3 in Santambrogio 2015.)